Antiparticle Contribution in the Cross Ladder Diagram for Two Boson Propagation in the Light-front.

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In the light-front milieu, there is an implicit assumption that the vacuum is trivial. By this " triviality " is meant that the Fock space of solutions for equations of motion is sectorized in two, one of positive energy k^- and the other of negative one corresponding respectively to positive and negative momentum k^+. It is assumed that only one of the Fock space sector is enough to give a complete description of the solutions, but in this work we consider an example where we demonstrate that *both* sectors are necessary.

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I. INTRODUCTION

In this work we explore the concept of covariant quantum propagator written down in terms of light front coordinates and obtain the propagator and Green's function in the light-front for a time interval x^+ where $x^+ = t + z$, is the light-front "time". In principle, this is equivalent to the canonical quantization in the light front [1, 2, 3, 4, 5, 6, 7]. Kogut and Soper [3] also makes use of this way of constructing quantities in the light front: starting with 4-dimensional amplitudes or equations, they integrate over $k^- = k^0 - k^3$, which plays the role of "energy" and corresponds to processes described by amplitudes or equations in "time" x^+. With this, the relative time between particles disappear and only the global propagation of intermediate state is allowed. The global propagation of the intermediate state is the "time" translation of the physical system between two instants x^+ and x'^+.

We specifically consider the case of two scalar bosons propagating forward in time. It has already been shown that for this case we can have exchange of two intermediate bosons via ladder diagrams or crossed ladder diagrams. For the ladder diagram we do not observe any new features, but for the crossed ladder diagram we have a crucial contribution coming from the other Fock space sector corresponding to the pair production. This is in fact a contribution that can be intuitively expected from the very nature of the crossed diagram, but that has never been noticed before or even considered. It is in this diagram that we can see manifestly the non-triviality of the light-front vacuum without any external perturbating field acting on the propagating bosons. We have already demonstrated that the non-trivial light-front vacuum also does manifest in the propagation of two scalar bosons when one of them is perturbed by an exernal background electromagnetic field [8].

In Classical Mechanics (CM), we characterize the state of a physical system by a point in the four dimensional space-time, (x_i, t), where x_i with $i = 1, 2, 3$ and t are respectively the space and the time coordinates. If we parametrize the time coordinate, we can describe the same system in the phase space (q, p), with $q = (q_1, q_2, ..., q_n)$ indicating collectively the degrees of freedom for such a system and $p = (p_1, p_2, ..., p_n)$ their respective conjugate momenta. Time evolution of the system is then described by a line either in the space-time or in the phase space $(q(t), p(t))$ which represents the set of values for all degrees of freedom and respective conjugate momenta at a given instant of time t. This time evolution is generated by Hamilton's equation. In this strictly deterministic theory, the state of the system at a given time is fixed by the initial conditions, as for example, at $t = 0$.

In Quantum Mechanics (QM), or wave mechanics, on the other hand, the state of a system is characterized by the wave function $\Psi(x, t)$, and its evolution is governed by Schrödinger's equation. The wave function must be known at each point of the space-time (x, t), so that it requires a continuous infinite values to be described, with the wave function taking the role of a field.

In a scattering process we have a wave function $\Psi(x, 0)$, where we fix the initial conditions at $t = 0$, coming into a point which has a force field or a particle, so that the typical question that arises is: Which is the wave function representation $\Psi(x, t)$ after the interaction with the scattering center?

In order to answer this question, we use the generalized Huygens principle [9]. If a wave function $\Psi(x, t)$ is known at a given time t, it is acceptable to assume that the wave function $\Psi(x', t')$ that emerges from the scattering centre located at (x, t) and propagates from position x to x' in a time $t' - t$, be proportional to the amplitude of the wave

function $\Psi(x,t)$. The proportionality contant is defined as $iG(x',t';x,t)$, so that we have in a mathematical notation:

$$\Psi(x',t') = i \int d^3x \, G(x',t';x,t) \, \Psi(x,t),$$

where $t' > t$.

Here $\Psi(x',t')$ defined at position x' and time t' is the wave function that emerges from the scattering centre. The quantity $G(x',t';x,t)$ is called the Green's function or propagator [9]. Knowing this G means to solve completely the scattering problem. In other words, knowing the Green's function is equivalent to solving of the Schrödinger's equation.

Now, is it possible to describe a physical system in any space-time hypersurface with initial conditions defined in a hypersurface different from $t = 0$? The answer to this question is positive and it was given long ago in 1949 by Dirac [10]. In his work, he proposes three distinct forms that could describe the dynamics of relativistic systems, two of which do not use time to describe the dynamical properties of simple systems. These different forms received different names: instant, point and front forms. The instant form is the usual relativistic dynamics described in terms of boundary conditions set at $t = 0$ while the front form uses the hyperplane of Minkowski space that contains the trajectory of light (light-cone).

II. FREE BOSON

The propagation of a free particle with spin zero in four dimensional space-time is represented by the covariant Feynman propagator

$$S(x^\mu) = \int \frac{d^4k}{(2\pi)^4} \frac{ie^{-ik^\mu x_\mu}}{k^2 - m^2 + i\varepsilon}, \tag{1}$$

where the coordinate x^0 represents the time and k^0 the energy. We are going to calculate this propagation in the light front, that is, for times x^+.

A point in the four-dimensional space-time is defined by the set of numbers (x^0, x^1, x^2, x^3), where x^0 is the time coordinate and $\mathbf{x} = (x^1, x^2, x^3)$ is the three-dimensional vector with space coordinates $x^1 = x$, $x^2 = y$ e $x^3 = z$. Observe that we adopt here the usual convention to take the speed of light as $c = 1$.

In the light front, time and space coordinates are mixed up and we define the new coordinates as follows:

$$x^+ = x^0 + x^3, \tag{2}$$
$$x^- = x^0 - x^3, \tag{3}$$
$$\vec{x}^\perp = x^1 \overrightarrow{i} + x^2 \overrightarrow{j}, \tag{4}$$

where $\overrightarrow{i}$ and $\overrightarrow{j}$ are unit vectors in the direction of x and y coordinates respectively.

The null plane is defined by $x^+ = 0$, that is, this condition defines a hyperplane that is tangent to the light-cone, the reason why many authors call the hypersurface simply by light-cone.

The initial boundary conditions for the dynamics in the light front are defined on this hyperplane. The axis x^+ is perpendicular to the plane $x^+ = 0$. Therefore a displacement of such hyperplane for $x^+ > 0$ is analogous to the displacement of a plane in $t = 0$ to $t > 0$ of the four-dimensional space-time. With this analogy, we recognize x^+ as the time in the null plane.

We make the projection of the propagator for a boson in time associated to the null plane rewriting the coordinates in terms of time coordinate x^+ and the position coordinates (x^- and $\vec{x}_\perp$). With these, the momenta are given by k^-, k^+ and $\vec{k}_\perp$, and therefore we have

$$S(x^+) = \frac{1}{2} \int \frac{dk_1^-}{(2\pi)} \frac{ie^{\frac{-i}{2}k_1^- x^+}}{k_1^+ \left(k_1^- - \frac{k_{1\perp}^2 + m^2 - i\varepsilon}{k_1^+}\right)}. \tag{5}$$

The Jacobian of the transformation $k^0, \vec{k} \to k^-, k^+, \vec{k}_\perp$ is equal to $\frac{1}{2}$ and k^+, $k_\perp$ are momentum operators.

Evaluating the Fourier transform, we obtain

$$\widetilde{S}(k^-) = \int dx^+ e^{\frac{i}{2}k^- x^+} S(x^+), \tag{6}$$

where we have used

$$\delta(\frac{k^- - k_1^-}{2}) = \frac{1}{2\pi} \int dx^+ e^{\frac{i}{2}\left(k^- - k_1^-\right)x^+},$$ (7)

and the property of Dirac's delta "function"

$$\delta\left(ax\right) = \frac{1}{a}\delta\left(x\right),$$ (8)

and we get

$$\widetilde{S}(k^-) = \frac{i}{k^+ \left(k^- - \frac{k_\perp^2 + m^2 - i\varepsilon}{k^+}\right)},$$ (9)

which describes the propagation of a particle forward to the future and of an antiparticle backwards to the past. This can be oberved by the denominator which hints us that for $x^+ > 0$ and $k^+ > 0$ we have the particle propagating forward in time of the null plane. On the other hand, for $x^+ < 0$ and $k^+ < 0$ we have an antiparticle propagating backwards in time.

The Green funtion in the light front $G(x^+)$ acting in the Fock space is defined as the probability amplitude of the transition from the initial state in the Fock space $|i\rangle$ to the final state $|f\rangle$. Its Fourier transform is sometimes called resolvent for a given Hamiltonian [11], however, here we call simply Fourier transform for the Green function or even Green function itself.

In the case of a free boson, the Green function for the propagation of a particle is defined by the operator

$$G_0^{(1p)}(k^-) = \frac{\theta(k^+)}{k^- - k_{on}^- + i\varepsilon} \; ;$$ (10)

where $k_{on}^- = \frac{k_\perp^2 + m^2}{k^+}$ is the energy of the particle. For the antiparticle propagation, we have:

$$G_0^{(1a)}(k^-) = \frac{\theta(-k^+)}{k^- - k_{on}^- - i\varepsilon} \; .$$ (11)

We can see that the difference between the Green functions in (10) and (11) for the propagator in the light front is the absence of the imaginary (complex) number i and of the factor of phase space k^+ which appears in (9).

The operator defined by (10) is the Green function of

$$\left(k^- - k_{on}^-\right)\left(G_0^{(1p)}(k^-) + G_0^{(1a)}(k^-)\right) = 1 \; .$$ (12)

The Feynman propagator is then rewritten as:

$$S(k^\mu) = \frac{i}{k^+}G_0^{(1p)}(k^-) - \frac{i}{|k^+|}G_0^{(1a)}(k^-) = \frac{i}{k^+(k^- - \frac{k_\perp^2 + m^2 - i\varepsilon}{k^+})} \; .$$ (13)

III.   GREEN FUNCTION OF TWO BOSONS

Our aim in this chapter is to study the two body Green function in the "ladder" approximation for the dynamics defined in the light front. Within this treatment, we are not going to deal with perturbative corrections that can be decomposed into one body problem.

Our interest is to define in the light front, the interaction between two bodies mediated by the interchange of a particle and obtain the correction to the two body Green function originated in this interaction.

For this purpose, we use a bosonic model for which the interaction Lagrangian is defined as:

$$\mathcal{L}_I = g\phi_1^*\phi_1\sigma + g\phi_2^*\phi_2\sigma,$$ (14)

where the bosons ϕ_1 and ϕ_2 have equal mass m and the intermediate boson, σ, has the mass m_σ. The coupling constant is g.

Taking from Dirac's idea [10] of representing the dynamics of a quantum system in the light front in time $x^+ = t+z$, we derive in this chapter the two body Green function or covariant propagator which describes the evolution of the

system from a hypersurface $x^+ = $ constant to another one. The Green function in the light front is the probability amplitude for a initial state in $x^+ = 0$ evolving to a final state in $x^+ > 0$, where the evolution operator is defined by the Hamiltonian in the light front [12]. Sometimes we call the resolvent $(Z - H)^{-1}$ as the Green function, too [7].

The two body Green function in the light front includes the propagation of intermediate states with any number of particles.

We start our discussion evaluating the second order correction to the coupling constant associated with the propagator. We define the matrix element for the interaction and so we obtain the correction to the Green function in the light front. Then we evaluate the correction to the Green function to the fourth order in the coupling constant, where we use the technique of factorizing the energy denominators, which is important to identify the global propagation of four bodies after the integration in the energies k^-. We discuss the generalization of this technique.

We show how to get the non perturbative Green function from the set of hierarchical equations for the Green function in the "ladder" approximation. This corresponds to the truncation of the Fock space in the light front, such that, only states with two bosons ϕ_1 and ϕ_2 are allowed, with no restriction as to the number of intermediate bosons σ. We discuss how to build a systematic approximation to the kernel of the integral equation for the two body Green function as a function of the number of particles in the intermediate Fock state and the power in the coupling constant. A consistent truncation can be carried out and in the lowest order, this brings to the Weinberg's equation for the bound state [13].

The propagator for the diagrams in interaction can be obtained of the functional generator (A10) and the are shown in the next sections.

IV.   ONE BOSON EXCHANGE

The perturbative correction to the propagator of two bodies in $O(g^2)$ comes from the interchange of a virtual intermediate boson, given by:

$$\Delta S_{g_S^2}^{(2)}(x^+) \; = \; (ig)^2 \int d\overline{x}_1^+ d\overline{x}_2^+ \, S_3(x^+ - \overline{x}_1^+) S_4(x^+ - \overline{x}_2^+)$$
$$\times S_\sigma(\overline{x}_1^+ - \overline{x}_2^+) S_1(\overline{x}_1^+) S_2(\overline{x}_2^+) \; . \tag{15}$$

The propagator of a particle $S_i(x^+)$ is defined in (5). The intermediate boson σ propagates between $\overline{x}_1^+$ and $\overline{x}_2^+$. The indices 1 and 2 in the particle propagators label initial states whereas 3 and 4 do so for the final states.

V.   TWO BOSON EXCHANGE (LADDER DIAGRAM):

In this section, we evaluate the perturbative correction to the propagator of two bosons up to the fourth order in the coupling constant, using the same method of the previous section. This corresction to the propagator constains terms obtained from the iteration of the Bethe-Salpeter equation in the "ladder" approximation up to the second order.

As we have a correction for the two boson propagator to the fourth order in the coupling constant, we have eigth propagators in the "ladder" diagram, two of which are propagators of the boson associated with the interation, identified as k_σ e $k_{\sigma'}$. We also make use of a lower index to define the particle momenta that propagate.

The perturbative correction to the Feynman propagator for two bodies up to order g^4 in the "ladder" approximation is given by:

$$\Delta S_{g^4}^{(2)}(x^+) \; = \; (ig)^4 \int d\overline{x}_1^+ d\overline{x}_2^+ d\overline{x}_3^+ d\overline{x}_4^+ \, S_5(x^+ - \overline{x}_3^+) S_6(x^+ - \overline{x}_4^+) S_{\sigma'}(\overline{x}_3^+ - \overline{x}_4^+) \times$$
$$S_3(\overline{x}_3^+ - \overline{x}_1^+) S_4(\overline{x}_4^+ - \overline{x}_2^+) \, S_\sigma(\overline{x}_1^+ - \overline{x}_2^+) S_1(\overline{x}_1^+) S_2(\overline{x}_2^+) \; . \tag{16}$$

Each propagator of the previous expression, is defined by (5). Therefore we substitute them in (16) and integrate in $\overline{x}_1, \overline{x}_2, \overline{x}_3$ and $\overline{x}_4$. With this, there emerges four Dirac deltas in $k_j^- (j = 1, ..., \sigma, \sigma')$, which correspond to the conservation of k^- in each vertex. So with these, eliminating four integrations in k^-, we are left with an exponencial $e^{\frac{-i}{2}\left(k_5^- + k_6^-\right)x^+}$. Evaluating the Fourier transform and integrating in x^+, this exponencial makes up another Dirac delta which later on, when performing integration in k_3^- results in the law of energy conservation for the system, given by,

$$K^- = k_5^- + k_6^- = k_1^- + k_2^- \; ,$$

FIG. 1: Exchange of two "crossed " σ bosons

where k_5^- and k_6^- are light front energies for the final state particles and k_1^- and k_2^- for the initial ones.

We note that in this ladder diagram there is no pair production and the two Fock spaces remain separate; the vacuum is trivial.

VI. CROSS LADDER DIAGRAM

This diagram brings about a new feature which was not present in the previous diagram just considered. Here we come across not only with all those diagrams that involve the propagation of information to future times, but note one particular diagram that bears pair production, i.e., there is one diagram which has intrinsic propagation of information to the past, thus mingling the two sectorized Fock spaces of solutions. This diagram, as far as we known, has not being considered in the literature before.

The correction to the two boson propagator coming from the process of two "crossed " σ bosons is represented by the Feynman diagram depicted in figure (1).

Explicitly this correction to the propagator is written down in terms of the one boson propagators and is given by the following equation

$$
\begin{aligned}
\Delta S_\times(x^+) \;=\; & (ig)^4 \int d\overline{x}_1^+ d\overline{x}_2^+ d\overline{x}_3^+ d\overline{x}_4^+ \, S_3(x^+ - \overline{x}_2^+) \times \\
& S_\sigma(\overline{x}_2^+ - \overline{x}_3^+) S_2(\overline{x}_2^+ - \overline{x}_1^+) S_6(x^+ - \overline{x}_4^+) \times \\
& S_\sigma(\overline{x}_4^+ - \overline{x}_1^+) S_5(\overline{x}_4^+ - \overline{x}_3^+) \times \\
& S_1(\overline{x}_1^+) S_4(\overline{x}_3^+),
\end{aligned}
\tag{17}
$$

and after Fourier transform we have

$$
\begin{aligned}
\Delta \widetilde{S}_\times(K^-) \;=\; & \frac{(ig)^4 \, i^8}{2^3 \, (2\pi)^3} \int dk^- dp^- dq^- \frac{dp^+ d^2 p_\perp}{k^+ p^+ (k-p)^+ (p-q)^+ (K-k-q+p)^+} \times \\
& \frac{1}{(K-q)^+ (q-p)^+ (k-p)^+} \times \\
& \frac{1}{\left(k^- - \frac{k_\perp^2 + m^2 - i\varepsilon}{k^+}\right)} \frac{1}{\left(p^- - \frac{k_\perp^2 + m^2 - i\varepsilon}{p^+}\right)} \times \\
& \frac{1}{\left(q^- - \frac{k_\perp^2 + m^2 - i\varepsilon}{q^+}\right)} \frac{1}{\left(K^- - k^- - \frac{(K-k)_\perp^2 + m^2 - i\varepsilon}{(K-k)^+}\right)} \\
& \frac{1}{\left(K^- - q^- - \frac{(K-q)_\perp^2 + m^2 - i\varepsilon}{(K-q)^+}\right) \left(k^- - p^- - \frac{(k-p)_\perp^2 + m_\sigma^2 - i\varepsilon}{(k-p)^+}\right)} \\
& \frac{1}{\left(K^- - k^- - q^- + p^- - \frac{(K-k-q+p)_\perp^2 + m^2 - i\varepsilon}{(K-k-q+p)^+}\right)} \\
& \frac{1}{\left(q^- - p^- - \frac{(q-p)_\perp^2 + m_\sigma^2 - i\varepsilon}{(q-p)^+}\right)},
\end{aligned}
\tag{18}
$$

FIG. 2: Crossed ladder diagrams time ordered in $x^+ > 0$.

For $K^+ > 0$, the regions of integration in p^+ which define the position of poles in the complex p^- are:

a) $0 < q^+ < p^+ < k^+ < K^+$

b) $0 < k^+ < p^+ < q^+ < K^+$

c) $0 < p^+ < q^+ < k^+ < K^+$

d) $0 < p^+ < k^+ < q^+ < K^+$

e) $0 < k^+ < q^+ < p^+ < K^+$

f) $0 < q^+ < k^+ < p^+ < K^+$

For regions "c" and "d" we use the method of partial fracioning twice to integrate in p^- [14]; for "a", "b", "e" and "f" this is not necessary; for regions "e" and "f" the integration in p^- vanishes. The diagrams for each of these regions are shown in figure (2).

After performing the analytic integrations in k^-, p^- and q^- we have

$$\Delta \widetilde{S}_\times(K^-) = \Delta \widetilde{S}_\times^a(K^-) + \Delta \widetilde{S}_\times^b(K^-) + \Delta \widetilde{S}_\times^c(K^-) + \Delta \widetilde{S}_\times^d(K^-) \tag{19}$$

where

$$\Delta \widetilde{S}_\times^a(K^-) = (ig)^4 \int \frac{i\, dp^+ d^2 p_\perp \theta(k^+ - p^+)\theta(p^+ - q^+)}{2k^+(K-k)^+ \left(K^- - \frac{k_\perp^2 + m^2 - i\varepsilon}{k^+} - \frac{(K-k)_\perp^2 + m^2 - i\varepsilon}{(K-k)^+}\right)}$$

$$\frac{1}{2p^+(k-p)^+(p-q)^+(K-k-q+p)^+} \times$$

$$\frac{i}{\left(K^- - \frac{p_\perp^2 + m^2 - i\varepsilon}{p^+} - \frac{(K-k)_\perp^2 + m^2 - i\varepsilon}{(K-k)^+} - \frac{(k-p)_\perp^2 + m^2 - i\varepsilon}{(k-p)^+}\right)} \times$$

$$\frac{i}{\left(K^- - \frac{(p-q)_\perp^2 + m_\sigma^2 - i\varepsilon}{(p-q)^+} - \frac{(k-p)_\perp^2 + m_\sigma^2 - i\varepsilon}{(k-p)^+} - \frac{(K-k)_\perp^2 + m^2 - i\varepsilon}{(K-k)^+} - \frac{q_\perp^2 + m^2 - i\varepsilon}{q^+}\right)} \times$$

$$\frac{i}{\left(K^- - \frac{q_\perp^2 + m^2 - i\varepsilon}{q^+} - \frac{(k-p)_\perp^2 + m_\sigma^2 - i\varepsilon}{(k-p)^+} - \frac{(K-k-q+p)_\perp^2 + m^2 - i\varepsilon}{(K-k-q+p)^+}\right)}$$

$$\frac{i}{2q^+(K-q)^+ \left(K^- - \frac{q_\perp^2 + m^2 - i\varepsilon}{q^+} - \frac{(K-q)_\perp^2 + m^2 - i\varepsilon}{(K-q)^+}\right)}, \tag{20}$$

and

$$\Delta\widetilde{S}^b_\times(K^-) = \Delta\widetilde{S}^a_\times(K^-)[k \leftrightarrow q],\tag{21}$$

shown in figures (2a) and (2b) respectively.

Regions "c" and "d" contribute to the propagator correction as

$$\Delta\widetilde{S}^c_\times(K^-) = (ig)^4 \int \frac{idp^+d^2p_\perp\theta(q^+-p^+)\theta(k^+-p^+)}{2k^+(K-k)^+\left(K^- - \frac{k_\perp^2+m^2-i\varepsilon}{k^+} - \frac{(K-k)_\perp^2+m^2-i\varepsilon}{(K-k)^+}\right)}$$
$$\frac{1}{2p^+(k-p)^+(p-q)^+(K-k-q+p)^+} \times$$
$$\frac{i}{\left(K^- - \frac{p_\perp^2+m^2-i\varepsilon}{p^+} - \frac{(q-p)_\perp^2+m_\sigma^2-i\varepsilon}{(q-p)^+} - \frac{(K-k-q+p)_\perp^2+m^2-i\varepsilon}{(K-k-q+p)^+} - \frac{(k-p)_\perp^2+m_\sigma^2-i\varepsilon}{(k-p)^+}\right)} \times$$
$$\widetilde{S}'^c_\times \frac{i}{2q^+(K-q)^+\left(K^- - \frac{q_\perp^2+m^2-i\varepsilon}{q^+} - \frac{(K-q)_\perp^2+m^2-i\varepsilon}{(K-q)^+}\right)},\tag{22}$$

where

$$\widetilde{S}'^c_\times = \frac{i}{\left(K^- - \frac{q_\perp^2+m^2-i\varepsilon}{q^+} - \frac{(k-p)_\perp^2+m_\sigma^2-i\varepsilon}{(k-p)^+} - \frac{(K-k-q+p)_\perp^2+m^2-i\varepsilon}{(K-k-q+p)^+}\right)} \times$$
$$\frac{i}{\left(K^- - \frac{k_\perp^2+m^2-i\varepsilon}{k^+} - \frac{(K-k-q+p)_\perp^2+m_\sigma^2-i\varepsilon}{(K-k-q+p)^+} - \frac{(q-p)_\perp^2+m^2-i\varepsilon}{(q-p)^+}\right)} \times$$
$$\frac{i}{\left(K^- - \frac{k_\perp^2+m^2-i\varepsilon}{k^+} - \frac{(q-p)_\perp^2+m_\sigma^2-i\varepsilon}{(q-p)^+} - \frac{(K-k-q+p)_\perp^2+m^2-i\varepsilon}{(K-k-q+p)^+}\right)} \times$$
$$\frac{i}{\left(K^- - \frac{p_\perp^2+m^2-i\varepsilon}{p^+} - \frac{(q-p)_\perp^2+m_\sigma^2-i\varepsilon}{(q-p)^+} - \frac{(K-q)_\perp^2+m^2-i\varepsilon}{(K-q)^+}\right)}.\tag{23}$$

The perturbative correction to the two boson propagator in Eq.(22) is represented by diagrams indicated in figure (2c). The correction represented by diagrams in figure (2d) is given by:

$$\Delta\widetilde{S}^d_\times(K^-) = \Delta\widetilde{S}^c_\times(K^-)\left[k \leftrightarrow K-k, p \leftrightarrow K-k-q+p, q \leftrightarrow K-q\right].\tag{24}$$

A. Antiparticle contribution

Next we deduce the antiparticle contribution to the crossed ladder diagram. This contribution happens for $p^+ < 0$ and $K^+ - k^+ - q^+ + p^+ < 0$. Let us analyse the first case, $p^+ < 0$.

The region for the "+" component momentum that allows the pole positioned in both hemispheres of the complex p^- plane, and therefore giving non-vanishing residue, are $-k^+ < p^+ < 0$ and $|p^+| + k^+ + q^+ < K^+$. So, the result for the momentum integration in "−" component for $0 < q^+ < K^+$ and $0 < k^+ < K^+$ which correspond to the non-vanishing results for integrations in k^- and q^- for $-k^+ < p^+ < 0$, is given by the diagram depicted in figure (3) and the result is given in (25):

$$\Delta\widetilde{S}^{ant}_\times(K^-) = \frac{(ig)^4}{8} \int dp^+d^2p_\perp \frac{\theta(k^+ + |p^+|)\theta(q^+ + |p^+|)}{k^+|p^+|q^+(|p|+k)^+(q+|p|)^+} \times$$
$$\frac{\theta(K^+ - |p^+| - q^+ - k^+)}{(K-k)^+(K-q)^+(K-|p|-q-k)^+} \times$$
$$\frac{i}{(K^- - K_0^-)}\frac{i}{(K^- - Q_0^-)}\frac{i}{(K^- - T^-)} \times$$
$$\frac{i}{(K^- - J_a^-)}\frac{i}{(K^- - T'^-)} +$$
$$+[k \leftrightarrow q] + [k \rightarrow K-k, q \rightarrow K-q] + [k \rightarrow K-q, q \rightarrow K-k],\tag{25}$$

FIG. 3: Pair creation process contributing to the crossed ladder diagram.

where

$$
\begin{aligned}
T^- &= \frac{(p-q)_\perp^2 + m_\sigma^2}{q^+ + |p^+|} + \frac{(K+p-q-k)_\perp^2 + m^2}{K^+ - k^+ - q^+ - |p^+|} + \\
&\quad + \frac{k_\perp^2 + m^2}{k^+}, \\
J_a^- &= \frac{q_\perp^2 + m^2}{q^+} + \frac{(K-k-q+p)_\perp^2 + m^2}{K^+ - k^+ - q^+ - |p^+|} + \\
&\quad + \frac{p_\perp^2 + m^2}{|p^+|} + \frac{k_\perp^2 + m^2}{k^+}, \\
T'^- &= \frac{(q-p)_\perp^2 + m_\sigma^2}{q^+ + |p^+|} + \frac{(K-k-w+p)_\perp^2 + m^2}{K^+ - k^+ - q^+ - |p^+|} + \\
&\quad + \frac{q_\perp^2 + m^2}{q^+}.
\end{aligned}
\tag{26}
$$

The four body propagator, J_a^-, (subindex a for antiparticle) has a propagation to past in the null-plane of an antiparticle with $p^+ < 0$. At instant $\overline{x}_2^+ > 0$ the pair particle-antiparticle is produced by the σ intermediate boson, then the antiparticle encounters a particle of momentum $k^+ > 0$, and is anihilated and the production of a σ boson with momentum $|p^+| + k^+ > 0$ which continues to propagate into the future of the null-plane.

VII. CONCLUSIONS

In our previous work we have demonstrated the pair production in the propagation of two scalar bosons in which one of them interacts with an external electromagnetic field. There, the propagation of a single boson interacting with the external field does not produce pair since this event is isolated; the second propagating boson which is the "spectator" boson (the one that sees the event happening) is crucial to the process to occur.

In the case of the ladder diagrams, the usual one does not produce pair too; only the crossed ladder diagram does. This is due to the fact that propagating intermediate bosons in crossed ladder diagrams has to have null-plane time ordered operators linked with causality playing its crucial role.

One very important observation that we pinpoint here is that in the light-front dynamics, the light-front coordinates x^+ and x^- are not simple 45^0 rotation of the usual four-dimensional space-time coordinates x^0 and x^3. It is rather a rotation constrained by the condition of light-likeness of the vector, i.e., $x^2 = 0$, which renders the components x^+ and x^- as being NOT linearly independent components; they do not serve as base vectors to generate the four-dimensional space-time. Moreover in the momentum integration, k^+ and k^- we have the same linearly dependence, here the difference being that the dispersion relation is the constraint, either $k^2 = 0$ for massless particles or $k^2 = m^2$ for massive particles (relation known as the Einstein relation).

We believe that most of the (if not all of them...) pathologies emerging in the light-front treatment of the physical processes up to now have been plagued by this simple misunderstanding and lack of perception. Our calculation has also been victimized by this lack of perception before. In reviewing the pathological behaviour of the process, for which no physical argumentation has been proposed to cure it, we were led to analyse more carefully the constraint defined by the dispersion relation. From it emerged the paradigm that in the light-front energy k^- and longitudinal momentum component k^+ are bound together in their signs; whether positive or negative, both of them bear the same sign as the other one. However, when we split the two and write energy in terms of momentum components, the longitudinal "+"-component of the momentum is carrying relevant physical information which can be lost by treating them as independent variables - which they certainly are not.

It is because of the above referred paradigm that up to now, most of the calculations done in the light front have always neglected or omitted the relevant contributions of the Fock space defined by the other sector, namely, the region of negative k^+.

APPENDIX A: GENERATING FUNCTIONAL

Using the concept of generating functional $Z[J]$, or vacuum-vacuum transition amplitude in the presence of an external source $J(x)$, we write:

$$Z[J] = \int \mathcal{D}\phi \exp\left\{i\int d^4x\left[\mathcal{L}(\phi) + J(x) + \frac{i\varepsilon}{2}\phi\right]\right\} \tag{A1}$$

$$\propto \langle 0,\infty\,|0,-\infty\rangle^J$$

where $\mathcal{L}(\phi)$ is the free scalar field Lagrangean (or Klein-Gordon field Lagrangean).

$$\mathcal{L}(\phi) = \frac{1}{2}\left(\partial_\mu\phi\partial^\mu\phi - m^2\phi^2\right) \tag{A2}$$

Using now the identity:

$$\partial_\mu\left(\phi\partial^\mu\phi\right) = \partial_\mu\phi\partial^\mu\phi + \phi\partial_\mu\partial^\mu\phi \tag{A3}$$

substituting this expression above into the Lagrangean (A2) which must be integrated over d^4x to calculate the generating functional. In this integration, the total derivative term (or divergent) does not contribute, due to Gauss ' theorem in four dimensions and considering that the field ϕ vanishes at infinity we have from (A3).

$$\int \partial_\mu\phi\partial^\mu\phi d^4x = -\int \phi\Box\phi d^4x \tag{A4}$$

We can then write the generating functional for the Klein-Gordon field without interaction, which we denote as $Z_0[J]$:

$$Z_0[J] = \int \mathcal{D}\phi \exp\left\{-i\int\left[\frac{1}{2}\phi\left(\Box + m^2 - i\varepsilon\right)\phi - J\phi\right]d^4x\right\} \tag{A5}$$

which after a simple calculation can be rewritten as:

$$Z_0[J] = N\exp\left[-\frac{1}{2}\int J(x)\Delta_F(x-y)J(y)\,dxdy\right] \tag{A6}$$

where dx and dy stand for d^4x and d^4y respectively and the normalization factor is

$$N = \int \mathcal{D}\phi \exp\left[-\frac{1}{2}\int \phi\left(\Box + m^2 - i\varepsilon\phi dx\right)\right]$$

We note that N does not depend on the external source $J(x)$ and since we are interested only in normalized transition amplitudes, we can write:

$$Z_0[J] = \frac{\left\{\mathcal{D}\phi\exp\left\{-i\int\left[\frac{1}{2}\phi\left(\Box + m^2 - i\varepsilon\right)\phi - J\phi\right]dx\right\}\right\}}{\left\{\int \mathcal{D}\phi\exp\left\{-i\int\frac{1}{2}\phi\left(\Box + m^2 - i\varepsilon\right)\phi dx\right\}\right\}}$$

$$Z_0[J] = \exp\left[-\frac{i}{2}\int J(x)\Delta_F(x-y)J(y)\,dxdy\right] \tag{A7}$$

where $\Delta_F(x)$ is the Feynmam propagator for the free scalar field and which has the following Fourier transformed representation:

$$\Delta_F(x) = \frac{1}{(2\pi)^4}\int d^4k\frac{e^{-ikx}}{k^2 - m^2 + i\varepsilon} \tag{A8}$$

and is the solution for the equation

$$\left(\Box + m^2 - i\varepsilon\right)\Delta_F\left(x\right) = -\delta^4\left(x\right) \tag{A9}$$

The Green 's functions are the expectation values of the time ordered field operators in the vacuum and can be written in terms of functional derivatives of the generating functional $Z_0\left[J\right]$, that is, $G\left(x_1, ..., x_n\right) = \langle 0|\, T\left(\phi\left(x_1\right)...\phi\left(x_n\right)\right)|0\rangle$ which is the n-point Green 's function of the theory, where

$$\langle 0|\, T\left(\phi\left(x_1\right)...\phi\left(x_n\right)\right)|0\rangle = \frac{1}{i^n}\left.\frac{\delta^n Z_0\left[J\right]}{\delta J\left(x_1\right)...\delta J\left(x_n\right)}\right|_{J=0}. \tag{A10}$$

Green's functions in field theory are extremely important because they are intimately related to the elements of the scattering matrix S through which we can calculate quantities measured directly from the experiments such as scattering processes where cross section for a particular reaction is measured, or decay of a particle into two or more where we can measure the half-lives of the particles involved, etc.

The propagator is associated to the Green's function equation as:

$$G(t - t') = -iS(t - t'). \tag{A11}$$

The Green 's function or the propagator describes completely the time evoltuion for the quantum system. In this present case we are using the propagator for "future times". We could also have defined the propagator "backwards" in time.

[1] S.D. Drell, D.J. Levy and J.-M. Yan, Phys. Rev. **D1** (1970) 1035.
[2] S.-J. Chang, R. G.Root e T.-M. Yan, Phys. Rev. **D7** (1973) 1133.
[3] J. B.Kogut and D. E.Soper, Phys. Rev. **D1** (1970) 2901.
[4] S.-J. Chang and T.-M. Yan, Phys. Rev. **D7** (1973) 1147; T.-M. Yan, *ibid.* **D7** (1973) 1780.
[5] N.E.Ligterink and B.L.G. Bakker, Phys. Rev. **D52** (1995) 5917; Phys. Rev. **D52** (1995) 5954.
[6] N.C.J. Schoonderwoerd and B.L.G. Bakker, Phys. Rev. **D57** (1998) 4965; Phys. Rev.**D58** (1998) 025013.
[7] S.J.Brodsky, H.-C.Pauli and S.Pinski, Phys.Rep.**301** (1998) 299.
[8] J.H.O.Sales and A.T.Suzuki, hep-th/0509116.
[9] W.Greiner and J.Reinhardt, "Quantum Electrodynamics", Springer, 1996.
[10] P.A.M. Dirac, Rev. Mod.Phys. **21** (1949) 392.
[11] A. Messiah, "Quantum Mechanics Vol.II ", North- Holland, Amsterdam, 1962.
[12] J.M.Namyslowski, Progress in Particle and Nuclear Physics **14** (1985) 49.
[13] S. Weinberg, Phys. Rev. **150** (1966)1313.
[14] J.H.O. Sales, T. Frederico, B.V. Carlson and P.U. Sauer, Phys. Rev. **C 61** (2000) 044003. Phys. Rev. **C 63** (20001) 064003.

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